14.4

Using the Liénard-Wiechert fields, discuss the time-averaged power radiated per unit solid angle in nonrelativistic motion of a particle with charge e, moving

(a) along the z axis with instantaneous position $z(t) = a \cos \omega_0 t$,

(b) in a circle of radius R in the x-y plane with constant angular frequency ω_0.

Sketch the angular distribution of the radiation and determine the total power radiated in each case.

14.7

A nonrelativistic particle of charge ze, mass m, and initial speed v_0 is incident on a fixed charge Ze at an impact parameter b that is large enough to ensure that the particle's deflection in the course of the collision is very small.

(a) Using the Larmor power formula and Newton's second law, calculate the total energy radiated, assuming (after you have computed the acceleration) that the particle's trajectory is a straight line at constant speed:

$$\Delta W = \frac{\pi z^4 Z^2 e^6}{3m^2 c^3 v_0} \frac{1}{b^3}$$

(b) The expression found in part a is an approximation that fails at small enough impact parameter. For a repulsive potential the closest distance of approach at zero impact parameter, $r_c = 2zZe^2/mv_0^2$, serves as a length against which to measure b. The approximation will be valid for $b \gg r_c$. Compare the result of replacing b by r_c in part a with the answer of Problem 14.5 for a *head-on* collision.

(c) A radiation cross section χ (with dimensions of energy times area) can be defined classically by multiplying $\Delta W(b)$ by $2\pi b\, db$ and integrating over all impact parameters. Because of the divergence of the expression at small b, one must cut off the integration at some $b = b_{\text{min}}$. If, as in Chapter 13, the uncertainty principle is used to specify the minimum impact parameter, one may expect to obtain an approximation to the quantum-mechanical result. Compute such a cross section with the expression from part a. Compare your result with the Bethe–Heitler formula [N^{-1} times (15.30)].

14.8

A swiftly moving particle of charge ze and mass m passes a fixed point charge Ze in an approximately straight-line path at impact parameter b and nearly constant speed v. Show that the total energy radiated in the encounter is

$$\Delta W = \frac{\pi z^4 Z^2 e^6}{4m^2 c^4 \beta} \left(\gamma^2 + \frac{1}{3}\right) \frac{1}{b^3}$$

This is the relativistic generalization of the result of Problem 14.7.

14.9

A particle of mass m, charge q, moves in a plane perpendicular to a uniform, static, magnetic induction B.

(a) Calculate the total energy radiated per unit time, expressing it in terms of the constants already defined and the ratio γ of the particles's total energy to its rest energy.

(b) If at time $t = 0$ the particle has a total energy $E_0 = \gamma_0 mc^2$, show that it will have energy $E = \gamma mc^2 < E_0$ at a time t, where

$$t \simeq \frac{3m^3 c^5}{2q^4 B^2} \left(\frac{1}{\gamma} - \frac{1}{\gamma_0}\right)$$

provided $\gamma \gg 1$.

(c) If the particle is initially nonrelativistic and has a *kinetic* energy T_0 at $t = 0$, what is its kinetic energy at time t?

(d) If the particle is actually trapped in the magnetic dipole field of the earth and is spiraling back and forth along a line of force, does it radiate more energy while near the equator, or while near its turning points? Why? Make quantitative statements if you can.

15.2

A nonrelativistic particle of charge e and mass m collides with a fixed, smooth, hard sphere of radius R. Assuming that the collision is elastic, show that in the dipole approximation (neglecting retardation effects) the classical differential cross section for the emission of photons per unit solid angle per unit energy interval is

$$\frac{d^2\sigma}{d\Omega\, d(\hbar\omega)} = \frac{R^2}{12\pi}\frac{e^2}{\hbar c}\left(\frac{v}{c}\right)^2 \frac{1}{\hbar\omega}(2 + 3\sin^2\theta)$$

where θ is measured relative to the incident direction. Sketch the angular distribution. Integrate over angles to get the total bremsstrahlung cross section. Qualitatively, what factor (or factors) govern the upper limit to the frequency spectrum?

15.4

A group of charged particles with charges e_j and coordinates $\mathbf{r}_j(t)$ undergo interactions and are accelerated only during a time $-\tau/2 < t < \tau/2$, during which their velocities change from $c\boldsymbol{\beta}_j$ to $c\boldsymbol{\beta}'_j$.

(a) Show that for $\omega\tau \ll 1$ the intensity of radiation emitted with polarization $\boldsymbol{\epsilon}$ per unit solid angle and unit frequency interval is

$$\frac{d^2I}{d\omega\, d\Omega} = \frac{1}{4\pi^2 c}|\boldsymbol{\epsilon}^* \cdot \mathbf{E}|^2$$

where

$$\mathbf{E} = \sum_j e_j\left(\frac{\boldsymbol{\beta}'_j}{1 - \mathbf{n}\cdot\boldsymbol{\beta}'_j} - \frac{\boldsymbol{\beta}_j}{1 - \mathbf{n}\cdot\boldsymbol{\beta}_j}\right)e^{-i\omega\mathbf{n}\cdot\mathbf{r}_j(0)/c}$$

(b) An ω^0 meson of mass 784 MeV decays into $\pi^+\pi^-$ and e^+e^- with branching ratios of 1.3×10^{-2} and 8×10^{-5}, respectively. Show that for both decay modes the frequency spectrum of radiated energy at low frequencies is

$$\frac{dI}{d\omega} = \frac{e^2}{\pi c}\left[\left(\frac{1+\beta^2}{\beta}\right)\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right] \simeq \frac{4e^2}{\pi c}\left[\ln\left(\frac{M_\omega}{m}\right) - \frac{1}{2}\right]$$

where M_ω is the mass of the ω^0 meson and m is the mass of one of the decay products. Evaluate approximately the *total* energy radiated in each decay by integrating the spectrum up to the maximum frequency allowed kinematically. What fraction of the rest energy of the ω^0 is it in each decay?